GEORGIA INSTITUTE OF TECHNOLOGY
OFFICE OF CONTRACT ADMINISTRATION

RESEARCH PROJECT INITIATION

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Date: October 13, 1975

Project Title: On An Algebraic Theory of Time-Varying Hereditary Systems With Applications to Control

Project No.: E-21-670

Principal Investigator: Dr. E. W. Kamen

Sponsor: U. S. Army Research Office, Research Triangle Park, N.C.

Agreement Period: From 10/1/75 Until 9/30/76

Type Agreement: Grant No. DAAG29-76-G-0048

Amount: $19,999 - ARO
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Reports Required:

Progress Report
Technical (When Justified)
Final Report

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Assigned to: Electrical Engineering

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RA-3 (9-75)

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GEORGIA INSTITUTE OF TECHNOLOGY
OFFICE OF CONTRACT ADMINISTRATION

SPONSORED PROJECT TERMINATION

Date: *April 28, 1977*

Project Title: *On An Algebraic Theory of Time-Varying Heredity Systems with Applications to Control*

Project No: *E-21-670*

Project Director: *Dr. E. W. Kamen*

Sponsor: *U.S. Army Research Office*

Effective Termination Date: 9/30/76

Clearance of Accounting Charges: 9/30/76

Grant/Contract Closeout Actions Remaining:

_ Final Invoice and Closing Documents

_ Final Fiscal Report

_ Final Report of Inventions

X Govt. Property Inventory & Related Certificate — submitted 29 Apr 77

_ Classified Material Certificate

_ Other ____________________

NOTE: ~~Continued by E-21-606~~

Assigned to: *EE* (School/Laboratory)

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CA–4 (3/76)

RESEARCH FINDINGS

1. General Aspects of Completed Research

The research has centered on the continued development of an algebraic theory of complex systems containing time-varying parameters and/or non-lumped elements such as pure and distributed time delays. During this past work period, significant new results have been obtained on

(1) representation of systems by first-order vector functional differential equations with time-independent or time-varying coefficients;

(2) realization of input/output differential (or difference) equations by first-order vector functional differential (or difference) equations;

(3) existence and construction of solutions of functional differential equations with initial functions;

(4) computable criteria for various dynamical properties such as reachability, controllability, and stabilizability;

(5) constructive procedures for designing state-feedback controllers to achieve some performance specification;

(6) constructive procedures for "reducing" system structure to simplified forms.

The potential applications of this work to U. S. Army problems are numerous, since many modern systems contain time delays and/or time-varying parameters that have a significant effect on system operation. For example, sizable time delays can result from

(1) communication links between subsystems located large distances from each other, as in telemetry between ground-based guidance systems and drones or missiles;

(2) reaction times or decision times of human operators in radar-tracking systems or aircraft control systems;

(3) on-line computer operations involving data storage or acquisition and data processing, such as smoothing over a large number of samples.

Systems with time-varying parameters also arise in many applications; for instance, communication systems employing the usual modulation techniques and multi-rate sampled-data systems or multi-rate digital filters.

The research should be of great use in guidance systems at the U. S. Army White Sands Missile Range. In particular, it could

(1) improve the accuracy of the control process, since there is no approximation of time delays resulting from computer processing or telemetry;

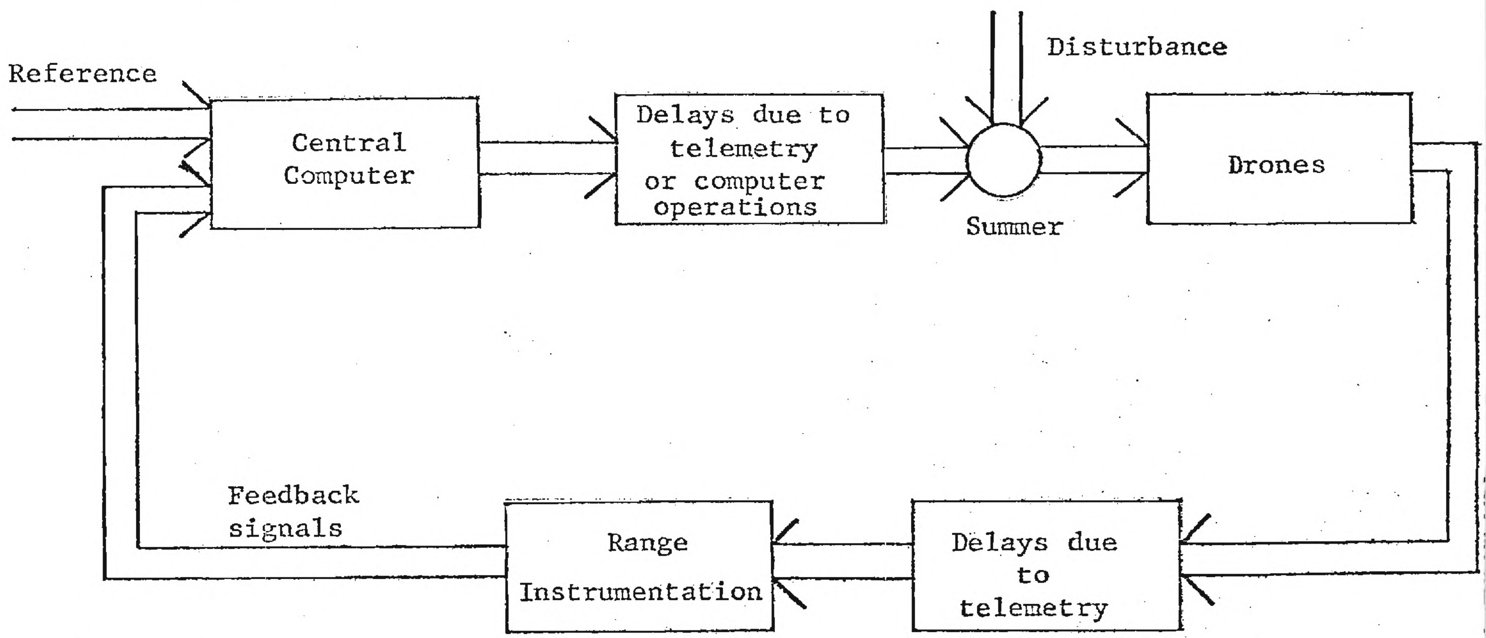
(2) reduce system dimension, as there is no need to approximate delays by n-pole low-pass filters (each n-pole filter approximation increases the system dimension by n);

(3) allow for the direct application of optimal control and optimal filter theory in the finite-dimensional framework, since via the algebraic theory we can first "reduce" system structure to the finite-dimensional case.

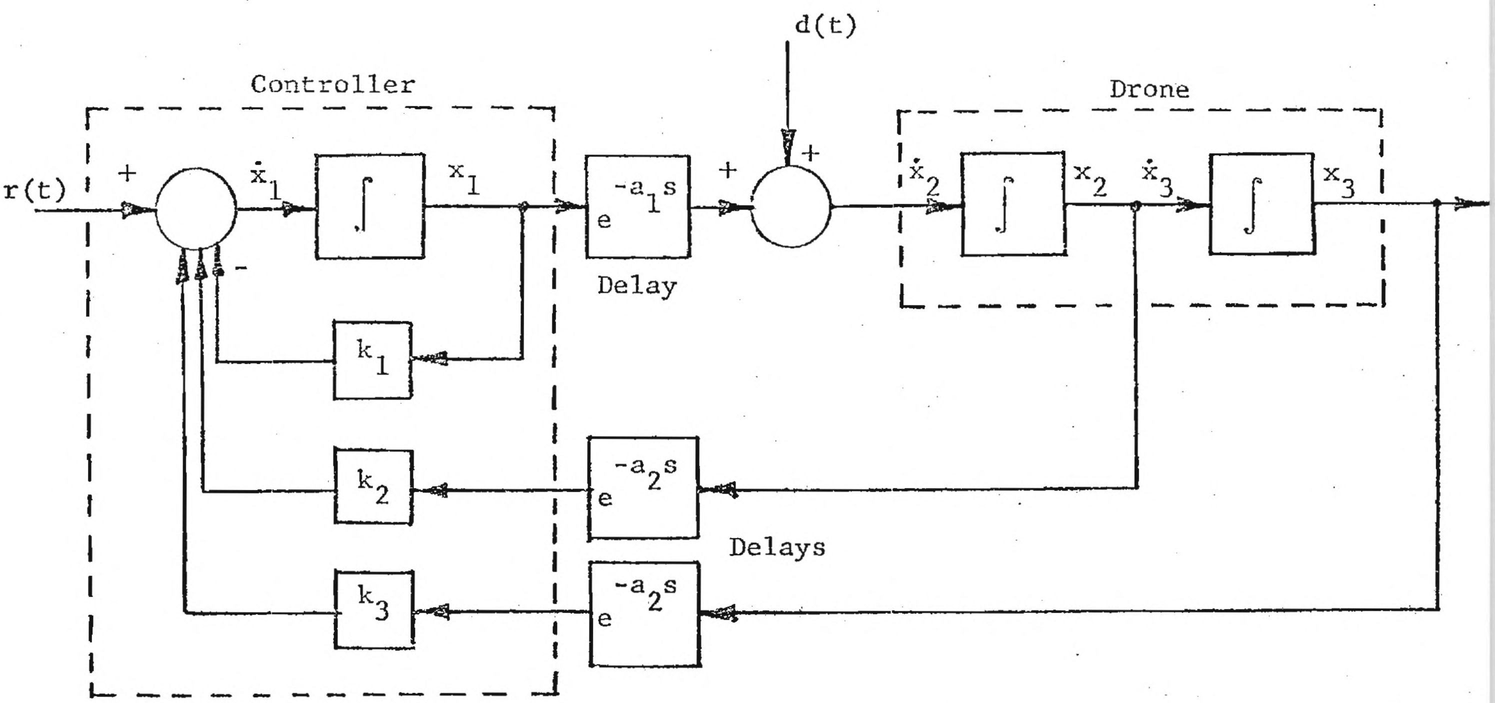
The application of the work to the guidance of missiles or drones is illustrated by the example which begins on the next page.

2. Example: Ground-Based Guidance of Drones

As a result of conversations with Dr. Alton Gilbert, who is at White Sands Missile Range, the author has learned that sizable time delays do arise in the ground-based guidance of drones and missiles. A very general block diagram of the control process is shown below.



In order to study the effects of the time delays, let us consider the following particular setup. The models for the controller and drone given below are not intended to be accurate representations of any existing system.



In this diagram, $r(t)$ is the reference signal, $d(t)$ is the disturbance, and k_1, k_2, k_3 are feedback gains. Let $X_3(s)$ (resp. $R(s), D(s)$) denote the Laplace transform of $x_3(t)$ (resp. $r(t), d(t)$). Then the reference transfer function, $T_R(s) = \frac{X_3(s)}{R(s)}$ with $D(s) = 0$, is given by

$$T_R(s) = \frac{e^{-a_1 s}}{s^3 + k_1 s^2 + e^{-(a_1+a_2)s}(k_2 s + k_3)}$$

and the disturbance transfer function, $T_D(s) = \frac{X_3(s)}{D(s)}$ with $R(s) = 0$, is given by

$$T_D(s) = \frac{s + k_1}{s^3 + k_1 s^2 + e^{-(a_1+a_2)s}(k_2 s + k_3)}$$

As a result of the $e^{-(a_1+a_2)s}$ component in the denominator of the transfer functions, in general it is very difficult to determine if a desired set of pole locations can be obtained for some choice of values of the feedback gains k_1, k_2, k_3. Thus it would be difficult to determine if a desired step response with a particular rise time, overshoot, settling time, etc. could be obtained by selecting k_1, k_2, k_3.

We could consider approximating $e^{-(a_1+a_2)s}$ by a one-pole low-pass filter with transfer function $1/(1 + (a_1+a_2)s)$, but this increases the order to 4, and it may happen that the optimal feedback gains k_1, k_2, k_3, computed with this approximation, invalidate the approximation.

Via the operator theory developed in [8], it was discovered that by placing a distributed delay in parallel with the feedback gain k_1, we can construct any desired denominator polynomial of the form $s^3 + b_2 s^3 + b_1 s + b_o$ where the b_i are real numbers. To explain this result, we first need to define distributed delays.

Let $w(t)$ be a locally-integrable function with $w(t) = 0$ for all t not contained in $[0,h]$, $h > 0$. An example of a distributed delay is a device with impulse response equal to $w(t)$. Thus if the input to the distributed delay is $u(t)$, the resulting zero state response $y(t)$ is given by

$$y(t) = \int_{t-h}^{t} w(t-\tau)u(\tau)d\tau$$

Now suppose that we place a distributed delay with impulse response $w(t)$ in parallel with the gain k_1. Letting $W(s)$ denote the Laplace transform of $w(t)$, we then have that the denominator of $T_R(S)$ and $T_D(s)$

is equal to

$$s^3 + (k_1 + W(s))s^2 + e^{-(a_1+a_2)s}(k_2 s + k_3)$$

With the distributed delay in the system, it turns out that the generation condition of Theorem 14 in [8] is satisfied, so that for any fixed real numbers b_2, b_1, b_o, there exist feedback gains k_1, k_2, k_3 and a distributed delay with impulse response $w(t)$ such that

$$s^3 + (k_1 + W(s))s^2 + e^{-(a_1+a_2)s}(k_2 s + k_3) = s^3 + b_2 s^2 + b_1 s + b_o$$

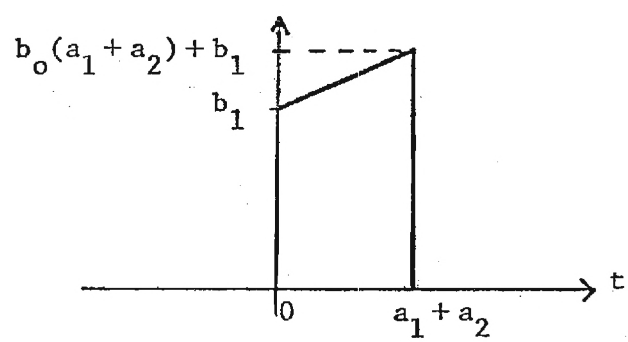
Given b_2, b_1, b_o and applying the procedure derived in [8], we have the following values for k_1, k_2, k_3 and $w(t)$:

$$k_1 = b_2$$

$$k_2 = b_o(a_1 + a_2) + b_1$$

$$k_3 = b_o$$

$w(t) =$



This result is very surprising in that we can "reduce" the system structure to the finite-dimensional case (that is, the denominator of the transfer function is a polynomial in s with real coefficients) without having to make the usual n-pole approximation of the time delays. Hence we can apply techniques available in the finite-dimensional setting, including the standard procedures derived from optimal control theory.

NOTE: Reference 8, mentioned above, is the paper "An Operator Theory of Linear Functional Differential Equations," by E. W. Kamen, submitted to the J. Differential Equations, May, 1976. The abstract of the paper is given below.

An operator theory, based on convolution rings, is developed for various classes of first-order vector functional differential equations of both the retarded and neutral type. The existence and construction of complete solutions is approached in a novel manner by incorporating initial data into the operator framework. Results are then obtained on exponential and asymptotic stability. The operator framework is also applied to the study of state feedback. Constructive results on stabilizability by feedback are given.